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The Planck Oscillators and the Black Hole Thermodynamics Correspondence

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Abstract In this brief note we show that a model of an array of oscillators at the Planck scale, correctly reproduces well known and also recently deduced results in the theory of Black Hole thermodynamics.

Keywords Planck scale · Oscillators · Black holes

1 Introduction

For some decades now, it has been recognized that the Planck Scale plays a fundamental role in the Universe, particularly if we are trying to unify Gravitation with Electromagnetism. There have been a few approaches like String Theory or Loop Quantum Gravity which have made some headway, though the desired unification is still elusive. We consider, in this context, the Planck Oscillator approach, which has given results that are consistent with known experiment and observation and described in detail in the following references [1–4]. What we would like to show in this note is that the approach leads to correct results in Black Hole Thermodynamics, results which were obtained by other considerations, and also to the concept of "Quantum of Area". We argue that a Black Hole can also be described as a collection of such oscillators.

2 The Planck Oscillators

Firstly we note that the Planck scale was defined by Max Planck himself more than a century ago, as is well known. He noticed that the following combination of fundamental constants

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define a length, mass and time interval, which therefore should also be in some sense fundamental.

$$
\begin{aligned}
l &= \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33}\ \text{cm} \\
m &= \sqrt{\frac{\hbar c}{G}} \sim 10^{-5}\ \text{gm} \\
t &= \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-42}\ \text{sec}
\end{aligned} \tag{1}
$$

It can be seen from the definition (1) [5] that l plays the role of the Compton length of the mass m as also the Schwarzchild radius of a Black Hole of the same mass (cf. [3] and references therein):

$$
l = \frac{\hbar}{2mc}, \qquad l = \frac{2Gm}{c^2} \tag{2}
$$

Indeed Rosen [6] had shown that a Planck mass of dimension of the Planck length is a Black Hole, a universe in itself. It may be mentioned that the Planck scale is the extreme scale where a quantum and a classical description converge [7, 8], or alternatively Gravitation and Electromagnetism converge, as we will see below.

Let us now consider a model discussed and described previously: An array of N particles, spaced a distance Δx apart, which behave like oscillators that are connected by springs. As is known we then have [3, 4, 9, 10] (cf. in particular Ref. [10])

$$
\begin{aligned}
r &= \sqrt{N \Delta x^2} \\
ka^2 &\equiv k\Delta x^2 = \frac{1}{2} k_B T
\end{aligned} \tag{3}
$$

where k_B is the Boltzmann constant, T the temperature, r the total extension and k is the spring constant given by

$$
\omega_0^2 = \frac{k}{m} \tag{4}
$$

$$
\omega = \left(\frac{k}{m} a^2\right)^{\frac{1}{2}} \frac{1}{2} = \omega_0 \tfrac{a}{r} \tag{5}
$$

It must be pointed out that (3) to (5) are general and a part of the well known theory referred to in [4, 9, 10]. In particular there is no restriction on the temperature T. m and ω are the mass of the particle and frequency of oscillation. In (4) ω_0 is the frequency of the individual oscillator, while in (5) ω is the frequency of the array of N oscillators, N given in (3).

We now take the mass of the particles to be the Planck mass and set $\Delta x \equiv a = l$, the Planck length as the mass and length are free parameters. We also use the well known Einstein–de Broglie relations that give quite generally the frequency in terms of energy and mass.

$$
E = \hbar\omega = mc^2 \tag{6}
$$

It may be immediately observed that if we use (4) and (3) we can deduce that

$$
k_B T \sim mc^2
$$

Independently of the above steps this agrees with the (Beckenstein) temperature of a Black Hole of Planck mass in the usual theory. Indeed as noted, Rosen [6] had shown that a Planck mass particle at the Planck scale can be considered to be a Universe in itself with a Schwarzchild radius equalling the Planck length.

Thus we have shown from the above theory of oscillators that an oscillator with the Planck mass and with a spatial extent at the Planck scale has the same temperature as the Beckenstein temperature of a Schwarzchild Black Hole of mass given by the Planck mass. We may reiterate that while (3) to (5) are valid generally, in the special case where the mass is taken to be the Planck mass and the distance a is taken to be the Planck length, we get a complete identification with the corresponding Schwarzchild Black Hole and the Beckenstein temperature. The above results can be obtained by a different route as described in [7]. We now rewrite (5) as (interchanging the roles of ω and ω_0)

$$\omega_0 = \frac{r}{a}\omega$$

Remembering that, quite generally, the frequency and mass are related by (6), i.e.,

$$\omega = \frac{mc^2}{\hbar}$$

we get on using (3)

$$\hbar\omega\left\langle \frac{l}{r}\right\rangle^{-1} \approx mc^2 \times \frac{r}{l} \approx Mc^2 = \sqrt{N}mc^2 \tag{7}$$

Generally, if an arbitrary mass M, as in (7), is given in terms of N Planck oscillators, in the above model, then we have from (7) and (3):

$$M = \sqrt{N}m \quad \text{and} \quad R = \sqrt{N}l \tag{8}$$

where R is the radius or extension of the object. We must stress the factor $\sqrt{N}$ in (8), arising from the fact that the oscillators are coupled, as given in (3). If the oscillators had not been coupled, or equivalently had not formed a coherent system, then we would have, for example, $M = Nm$ or $R = Nl$ instead of (8). Using the fact that l has been chosen to be the Schwarzchild radius of the mass m, this gives immediately,

$$R = 2GM/c^2$$

This shows that if an arbitrary mass M consists of N coherent Planck oscillators as above, and specifically equation (8), then its radius R is given by the above expression, which is its Schwarzchild radius. In other words, such an object shows up as a Schwarzchild Black Hole. It must be emphasized that the expression for R follows from the theory of oscillators, specifically equation (8) and shows that it is identical to the Schwarzchild radius for the same mass M. We have merely used the known equivalence of the Planck length and Schwarzchild radius for the Planck mass. We next use the fact that, as shown by Cercignani [11, 12] for Planck oscillators not to be chaotic we require a maximum frequency $\omega_{\max}$ such that

$$G\hbar\omega_{\max}^2 = c^5 \tag{9}$$

For frequencies greater than ω_0, the system is totally unphysical. Using (4), (3) and (9) we get

$$k_B T = m\omega_{\max}^2 l^2 = \frac{c^5/G\hbar}{ml^2} = \frac{\hbar c^3}{Gm},$$

remembering that l by (2) is the Compton wavelength. That is we get

$$k_B T = \frac{\hbar c^3}{Gm} \tag{10}$$

Equation (10) is the well known Beckenstein temperature formula valid for a Black Hole of arbitrary mass but derived here for the Planck mass.

Can we now generalize (10) to the case of a Black Hole of arbitrary mass, as in the original Beckenstein formula but using only the characterization of the Black Hole in terms of Planck oscillators, as above? This is what we will do. In fact to a Black Hole of mass M characterized in terms of N oscillators as in (8), we associate a Black Hole temperature defined by

$$\bar{T} = \frac{T}{\sqrt{N}},$$

where T is given in (10). Using this with (8) in (10) we immediately get

$$k_B \bar{T} = \frac{\hbar c^3}{GM} \tag{11}$$

Equation (11) which is the analogue of (10) is the required result. After this identification, we next use the following known relations for a Schwarzchild Black Hole [13]:

$$dM = TdS, \quad S = \frac{kc}{4\hbar G} A \tag{12}$$

where T is the Black Hole temperature, now identified with (11), S the entropy and A is the area of the Black Hole. The area is given by, using (8)

$$A = Nl^2 \tag{13}$$

because, this area is $\sim R^2$. Alternatively this shows that there are N elementary areas l^2 forming the Black Hole. Indeed this defines the basic quantum of area of quantum gravity approaches and is in pleasing agreement with the result of Baez deduced from a different quantum gravity consideration [14].

Using (8), (10) and (13), we can easily see that (12) is valid for the mass M given by (8) or (7).

This completes the identification of Black Holes characterized by coherent Planck oscillators, with the conventional Hawking–Beckenstein theory.

3 Conclusion

Over the past few years, the Planck oscillator model has been discussed at length (cf. Refs. [1–4] and several references therein). This scenario models an underpinning for the universe, consisting of oscillators at the Planck scale: Thus oscillators are the analogue of a system of particles connected by springs or an array of atoms in a crystal. The mass of each particle m, is the Planck mass $\sim 10^{-5}$ gm while the distance between them is the Planck length l, $\sim 10^{-33}$ cm. What we have shown in this paper is that such a system of Planck oscillators can also model Black Holes—there is a complete correspondence between Black

Hole Thermodynamics and a system of N such Planck oscillators, where the mass of the Black Hole

$$M = \sqrt{N}m$$

and its size is given by,

$$R = \sqrt{N}l$$

Pleasingly also in this model the area of the Black Hole as given in (13), is in agreement with the recent Quantum Gravity result.

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